Effective black holes from non-Riemannian vortex acoustics in ABC flows

by

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Abstract

It is shown that the existence of effective black holes in non-Riemannian vortex acoustics is not forbidden under certain constraints on acoustic torsion vector. In Arnold-Beltrami-Childress (ABC) flows only the spatial part acoustic torsion vertical vanish, which allows us to consider constraints on acoustic torsion to obtain artificial non-Riemannian black hole solutions in ABC flows. An explicity acoustic black hole solution is given for irrotational ABC flows, This solution is similar to Visser acoustic metric for a vortex flow. Actually a particular form of this acoustic metric is given for ABC flows. Effective BHs with acoustic torsioned vortex flows in effective spacetime. The Ricci scalar computed is the same as that of a string singularity and the black-hole ergoregion coincides with the event horizon.**PACS numbers: 02.40.Hw:differential geometries.**

I Introduction

Previously the author [1] has built a non-Riemannian vortex acoustics theory of effective gravity [2] which generalised the Unruh Visser [3, 4] effective theory of artificial black holes [4] in other laboratory frameworks as optical and acoustic media as well as more recently plasma matter [5]. These acoustic black holes are effective analogue pseudo-Riemannian metrics given by the homogeneous wave equation from linearised Euler flows or maximum Navier-Stokes. Non-Riemannian vortex acoustic metrics, of course, cannot be reduced to Unruh-Visser (UV) under no constraint acoustic BHs when Cartan torsion vector vanishes therefore expression (18) in reference [1] is not wrong and only the expression for the time component of acoustic torsion trace is wrong. In this paper besides of correcting this mistake we work it out the its consequences. First it is shown that even the non-vorticity flows sonic BHs may exist in non-Riemannian effctive spacetime. A non-diagonal sonic metric is obtained in this effective spacetime. Constraining the equations in non-Riemannian flows [5] where the torsion fluctuation is orthogonal to the perturbed flow, it is shown that Riemannian sonic black holes can be also obtained. ABC flow and its effective spacetime are also considered.

II Acoustic Black-holes in effective Riemann-Cartan ABC flows

In this section we present the brief results described above. The non-Riemannian acoustic effective geometry endowed with Cartan torsion , usually called Riemann-Cartan effective spacetime. These equations which shall be used to build the generalized acoustic BHs, are given by the force equation [1]

$$\rho[\partial_t \mathbf{v} + (\mathbf{v}.\nabla)\mathbf{v}] = -\nabla p \tag{II.1}$$

$\mathbf{v}$ is only partially irrotational

$$\mathbf{v}_1 = \nabla\psi_1 \tag{II.2}$$

which yields

$$\vec{\Omega}_1 = \nabla\times\nabla\psi_1 = 0 \tag{II.3}$$

where Ω_1 is the vorticity fluctuation according to the rules the fields fluctuations [4]

$$p = p_0 + \epsilon p_1 \tag{II.4}$$

$$\psi = \psi_0 + \epsilon\psi_1 \tag{II.5}$$

$$\rho = \rho_0 + \epsilon\rho_1 \tag{II.6}$$

$$\vec{\Omega} = \vec{\Omega}_0 + \epsilon\vec{\Omega}_1 \tag{II.7}$$

and the conservation mass equation

$$\partial_t\rho + \nabla.(\rho\mathbf{v}) = 0 \tag{II.8}$$

Along with the barotropic equation of state

$$p = p(\rho) \tag{II.9}$$

where p is the pressure, substitution of the above fluctuations into the evolution flow equations one obtains the generalised Visser equation as

$$\partial_t[c^{-2}{}_{sound}\rho_0(\partial_t\psi_1+\mathbf{v}_0.\nabla\psi_1)]+\partial_t[c^{-2}{}_{sound}\rho_0\delta+\alpha]] = \nabla.[\rho_0\nabla\psi_1-c^{-2}{}_{sound}\rho_0\mathbf{v}_0(\partial_t\psi_1+\mathbf{v}_0.\nabla\psi_1)+\alpha] \tag{II.10}$$

from the RC torsion one obtains the wave equation

$$\Box\psi_k = -T^l{}_{kl}\psi^k \tag{II.11}$$

where $T^l{}_{kl}$ is the torsion tensor trace. From the above equations one obtains

$$T^0 = -\frac{\rho_0}{{c_{sound}}^2} \tag{II.12}$$

and [1]

$$\mathbf{T} = \frac{\rho_0}{{c_{sound}}^2}\Omega_0 \times \mathbf{v}_0 \tag{II.13}$$

actually formula (II.12) corrects previous results in reference [1]. Actually to have the torsion trace compute as above one has to impose on the unperturbed flow before fluctuation, because the RHS of equation (II.10) can be expressed as

$$(\nabla.\mathbf{v}_0)\alpha - (\mathbf{v}_0.\nabla)\alpha \tag{II.14}$$

and only the second in this expression contributes to acoustic torsion. Therefore to be compatible with Unruh metric plus acoustic torsion the first term in (II.14) has to vanish and this corresponds to the situation that either

$$\nabla.\mathbf{v}_0 = 0 \tag{II.15}$$

which means incompressibility and the second condition namely that α vanishes means. Of course this condition shows that when the unperturped fluid is an ABC flow, the sonic BHs are actually Riemannian, but since this is impossible due the fact that T^0 cannot vanish, the sonic BH in ABC unperturbed flow has to be incompressible in this effective spacetime. Now let us apply these results in ABC flows. In these Beltrami like flow the vorticity is proportional to the own velocity flow as described in the equation

$$\vec{\Omega}_0 = \nabla \times \mathbf{v}_0 = \lambda \mathbf{v}_0 \tag{II.16}$$

where λ is constant. Thus when only the unperturbed part of the flow is ABC's fluid one immeadiatly notes from expression (II.13) that acoustic torsion spatial part vanishes. However , the time component of torsion trace build as a vector, does not vanish and

effective spacetime is still non-Riemannian. Now by dropping the exigence of ABC flows and considering irrotational incompressible flows one may write

$$\nabla.\mathbf{v}_0 = \nabla^2\psi_0 = 0 \tag{II.17}$$

This can have a solution

$$\psi_0 = \frac{A}{r} \tag{II.18}$$

and by the effective metric the Riemannian acoustic metric given by

$$g^{00} = -\frac{1}{c^2{}_{sound}\rho_0} \tag{II.19}$$

$$g^{0j} = -\frac{1}{c^2{}_{sound}\rho_0}v^j{}_0 \tag{II.20}$$

$$g^{ij} = \frac{1}{c^2{}_{sound}\rho_0}(c_{sound}(\delta^{ij} - {v_0}^i{v_0}^j)) \tag{II.21}$$

one obtains a Riemannian black hole solution as

$$g^{0j} = \frac{1}{c^2{}_{sound}\rho_0}\frac{A}{r^2}\delta^{j0} \tag{II.22}$$

and

$$g^{ij} = \frac{1}{c^2{}_{sound}\rho_0}(c_{sound}(\delta^{ij} - \frac{A^2}{r^4}\delta^{ij})) \tag{II.23}$$

The Riemannian line element is given by

$${ds_{2+1}}^2 = -({c_{sound}}^2 - \frac{A^2}{r^2})dt^2 + (dr - \frac{A}{r}dt)^2 + r^2d\theta^2 \tag{II.24}$$

which possesses a curvature Ricci scalar like

$$R_{ABC} = -\frac{4{c_{sound}}^2A^2}{r^4} \tag{II.25}$$

which coincides with the string metric curvature scalar computed by Visser [6]. In our case the ergo-region coincides with the event horizon since

$$r_{ergo-region} = \frac{A}{c^2{}_{sound}} \tag{II.26}$$

Therefore one finally obtains a new solution of sonic Riemannian black holes. A non-Riemannian solution can be given by considering a solution such as $\mathbf{v}_0 = constant$ which reduces the torsion trace to a zero spatial part and the flow is irrotational. Thus one concludes that it is possible to have a stationary unperturbed flow without having a vanishing torsion trace, which means that non-Riemannian sonic black holes exists even for non-vortex flow.

III Conclusions

Recently a deep connection between vortex flows and non-Riemannian artificial acoustic BHs and the by Garcia de Andrade [1]. In this report one shows that this sonic non-Riemannian BHs may exists even in the absence vorticity. Other models in Einstein-Cartan gravity [5] may be suitable for finding other solutions of acoustic BHs.

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